Exercise 3.3.7

Show that e^x is the sum of an even and an odd function.

Solution

$$\begin{aligned} e^x &= \frac{e^x}{2} + \frac{e^{-x}}{2} + \frac{e^x}{2} - \frac{e^{-x}}{2} \\ &= \frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2} \\ &= \cosh x + \sinh x \end{aligned}$$

$\cosh x$ is an even function because $\cosh(-x) = \cosh x$, and $\sinh x$ is an odd function because $\sinh(-x) = -\sinh x$. Therefore, e^x is the sum of an even and an odd function.